

# NCPC 2024

Presentation of solutions



October 5, 2024

## Problems prepared by

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## C — Composed Rhythms

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Given an integer  $n \leq 10^6$ , write it as a sum of twos and threes.

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Statistics at 4-hour mark: 394 submissions, 209 accepted, first after 00:02

# K — Knitting Pattern

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Figure out how to centre a knitting pattern on a sweater.

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- 3 The only exception is if we fit one more pattern exactly in the middle on the back, so when the leftover space is exactly  $p$  or equivalently when  $p$  divides  $n$ . In this case the answer is zero instead.

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Statistics at 4-hour mark: 687 submissions, 152 accepted, first after 00:04

# A — Avoiding the Abyss

## Problem

Given a starting point, target point, and one point in a rectangular swimming pool, find a way to walk from the start to the target without intersecting the pool.

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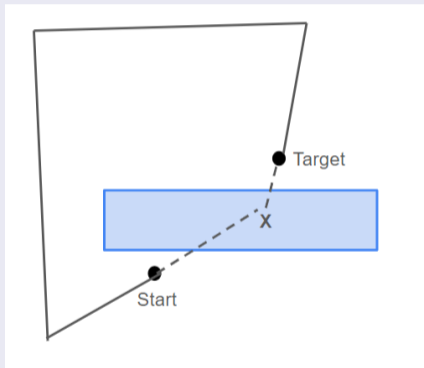
You can't intersect the pool if you are outside of  $-10^4 \leq x, y \leq 10^4$

### ② Observation 2

You can't intersect the pool while walking away from the pool point.

# A — Avoiding the Abyss

## Solution



Statistics at 4-hour mark: 412 submissions, 126 accepted, first after 00:16



# I — Infinite Cash

## Problem

Given the starting money, salary and salary frequency of a fiscally unsound man, find out how long he has before he goes broke (if ever).

## Solution

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- 2 The other is to do things more by hand. Let  $b(x)$  be the number of binary digits in  $x$ . Then to check if he goes broke before his first payment, check whether  $b(m) < d$ . To check whether he never goes broke, check whether  $d \leq b(s)$ . Then if  $b(d) > 12$  just print  $b(m)$ . Finally simulate the salary periods one by one and print when he runs out of money.

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Statistics at 4-hour mark: 575 submissions, 81 accepted, first after 00:12

## D — Double Deck

### Problem

Solve the longest common subsequence problem when the number of occurrences of any specific value is  $\leq 15$ .

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Statistics at 4-hour mark: 214 submissions, 23 accepted, first after 00:10

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Compute the faces of the provided planar graph, and find the areas (squared) of all faces (except the outer face).

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- 3 Repeat the process for  $vu$  to find the right-hand face.
- 4 Detect the outer face (the leftmost vertex (break even on  $y$  coordinate) and its neighbor with angle highest  $\leq 90^\circ$ )
- 5 Use the shoelace algorithm for computing the area of each face/polygon.

Statistics at 4-hour mark: 61 submissions, 16 accepted, first after 00:50

# J — Jungle Game

## Problem

You are given  $N$  forbidden pairs  $(P_i, S_i)$  such that  $1 \leq P_i, S_i \leq 2N$ . Find  $N$  different pairs  $(p_i, s_i)$  satisfying  $1 \leq p_i, s_i \leq N$ , such that  $(p_i + p_j, s_i + s_j)$  is never forbidden.

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## Solution

### 1 Case 1.

If there is an even number  $S$  such that  $(P, S)$  is never forbidden, then you can answer  $(1, S/2), (2, S/2), \dots, (N, S/2)$ .

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If there is an even number  $P$  such that  $(P, S)$  is never forbidden, then you can answer  $(P/2, 1), (P/2, 2), \dots, (P/2, N)$ .

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### 3 Case 3:

Otherwise, both lists  $P_i$  and  $S_i$  are permutations of  $2, 4, 6, \dots, 2N$ .

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- 1 If  $N = 1$ , it is impossible.
- 2 Otherwise, find the unique forbidden point  $(2, S)$ . Add the points  $(2, S/2), (3, S/2), \dots, (N, S/2)$  to the solution. We now need to find just one more point.

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- 3 If  $S/2$  is even, then either  $(1, 1)$  or  $(2, 1)$  will work. Otherwise, either  $(2, 1)$  or  $(2, 2)$  will work. Try all four.
- 4 Running time:  $O(N)$ .

## Heuristic solutions

The intended solution is  $O(N)$ , but checking the answer takes  $O(N^2)$ . Unfortunately, this means that various  $O(N^2)$  heuristics can pass as well.

- 1 Iterate through every pair  $(p, s)$  in a **random** order.
- 2 For each pair, check if it can be added to the answer.

**Question:** If the order is randomized, does the above solution work with high probability (or should we have made better test data)?

Statistics at 4-hour mark: 35 submissions, 8 accepted, first after 01:23



## G — Guessing Passwords

### Problem

Given a bipartite graph where all but the last node in the left half have the same degree, find an edge-colouring such that each colour touches exactly one node in the left half or only the last node in the left half.

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Statistics at 4-hour mark: 11 submissions, 1 accepted, first after 03:15

## B — Baseball Court

### Problem

Given bounds  $a, b$  how many partitions  $\rho$  of  $n$  satisfy  $|\rho| \leq b$ ,  $\max \rho \leq a$  and  $\rho_i + i$  is constant across all indices  $i$  such that  $\rho_i \neq \rho_{i+1}$ .

### Solution

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- 3 The problem can now be solved using dynamic programming.

### Solution

- ① Consider the maximum index  $k$  such that  $\rho_1 = \rho_k$  and  $\rho_k \neq \rho_{k+1}$ . Then  $\rho_k + k = d + 1$ . Then by cutting off these front values we remove  $k$  values and a sum of  $k(d + 1 - k)$ . This gives us the recurrence

$$f(n, d) = \sum_{k=1}^d f(n - k(d + 1 - k), d - k)$$

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- ② This is  $\mathcal{O}(n^3)$  which is not quite good enough. But  $f(n - k(d + 1 - k), d - k)$  has the first argument  $< 0$  for all but the first and last  $\sqrt{n}$  terms. So we can skip those and get a time complexity of  $\mathcal{O}(n^{2.5})$ .

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- ③ Then we simply subtract all partitions that have  $|\rho| > a$  or  $\max \rho > b$ , then add back all the ones that violate both to get the right answer by inclusion-exclusion.

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Statistics at 4-hour mark: 14 submissions, 0 accepted, first after ????

## E — Elapid Errands

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Given  $N = 20$  random points in a grid, visit all of them in order while never intersecting yourself.

### Solution

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### Solution

- 1 Greedily walk to each point? You will probably intersect yourself.
- 2 Also avoid visiting the same point twice? The plane will probably get divided into disjoint regions where future points can't be visited.

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- 1 Greedily walk to each point? You will probably intersect yourself.
- 2 Also avoid visiting the same point twice? The plane will probably get divided into disjoint regions where future points can't be visited.
- 3 Main idea: Try to make a snake region that doesn't contain any holes. This way, points will not become unreachable.



### Solution

For each point  $P$  you want to visit:

- 1 Find a point  $Q$  on the snake boundary that is as close as possible to  $P$ .
- 2 Walk along the boundary of the snake to  $Q$ .
- 3 Greedily walk from  $Q$  to  $P$ . You will not intersect yourself since  $Q$  was as close as possible to  $P$ .

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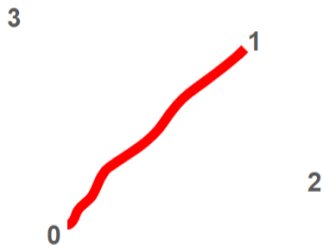
3

1

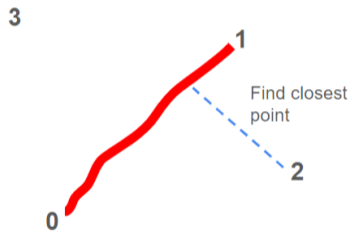
2

0

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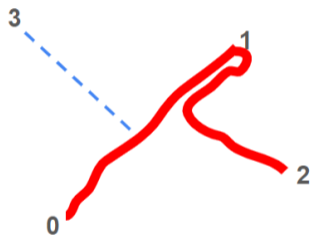
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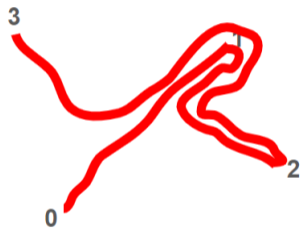
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### Issues

- 1 In the third step, it is possible that you accidentally pass too close to a future point. Put a "danger zone" around every future point so that you don't step on them too early.
- 2 The snake region will probably create 1x1 holes, some care is needed to make sure you don't walk into a dead end.

Statistics at 4-hour mark: 25 submissions, 0 accepted, first after ??:??



## Problem

Given a program that prints every unique substring of a string along with its number of occurrences, find out how many copies of each non-whitespace character that program would print for a given string.

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## Solution

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- 2 We now solve the characters and numbers separately.

## Problem

Given a program that prints every unique substring of a string along with its number of occurrences, find out how many copies of each non-whitespace character that program would print for a given string.

## Solution

- 1 To solve this we need suffix arrays and segment trees. We start by constructing the suffix array and its longest common prefix array.
- 2 We now solve the characters and numbers separately.
- 3 For the letters we use a lazy propagation segment tree that allows for a range update where the first element is incremented by  $b$ , the next by  $a + b$ , the third by  $2a + b$  and so on.

## Solution

- 1 The segment tree will keep track of the number of occurrences of each character in the input string as we iterate over the suffix array. Let  $S$  be the  $i$ -th element of the suffix array and  $L$  be the  $(i - 1)$ -st element of the longest common prefix array.

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- 2 Then  $n - S - L$  new prefixes start at position  $S$ . The first  $L$  possible substrings starting at  $S$  are duplicates. So we increment the segment tree by  $n - S - L$  at positions  $S$  through  $S + L - 1$ . Then we increment the segment tree by  $n - S - L$ ,  $n - S - L - 1$ ,  $n - S - L - 2$ , and so on at positions  $S + L$  through  $n - 1$ . At the end of this all we can read off the number of occurrences of each character from the segment tree.

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Statistics at 4-hour mark: 42 submissions, 0 accepted, first after ??:??

Results!